A Flexible and Low-Complexity Locally Erasure Recovery Scheme

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August 8, 2017
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• Large-scale storage system needs to be able to recover or tolerate multiple failed or unavailable disk/SSD/NAND dies

• Goal of traditional erasure coding schemes: minimize redundancy
  – An \((n,k)\) maximum distance separable (MDS) code has \(t = d_{\text{min}} - 1 = n - k\)

• Traditional erasure codes
  – XOR-single parity \((t = 1)\)
  – RAID-6 (EVENODD, Row-Diagonal Parity (RDP), extended RS...) \((t = 2)\)
  – Reed-Solomon (RS) codes \((t \geq 2)\)

• Need to read \(k\) symbols in order to recover any erasure
Erasure Codes with Local Recovery

• Distributed storage needs erasure codes accessing much less than \( k \) symbols for recovery (possible if actual erasure number \( < t \))
  – Lower network traffic
  – Reduced recovery latency
  – Better data availability

• Recent regenerating and local erasure recovery codes
  – Modified EVENODD and RDP (have constraints on \( n \) and \( k; t = 2 \))
  – Rotated RS (\( t = 2, 3 \))
  – Piggybacking (locality depends on failure indices)
  – Codes based on partitions of finite fields (high en/decoder complexity)
  – Sparse parity columns in the generator matrix, each parity covers overlapping data symbols (worse locality for data symbol recovery)

• Our new scheme for local recovery
  – Not much constraint on \( n,k,t \)
  – Easy tradeoff on locality and redundancy
  – Allows unequal protection
  – Low-complexity en/decoder
MDS Codes With Systematic Parity Check Matrices

• For a linear block code, $x$ is a code word iff $Hx^T = 0$
• $H = [A|I] \rightarrow x$ consists of data symbols followed by parities

$$H = \begin{bmatrix}
1 & 1 & \ldots & 1 & 1 & 0 & \ldots & 0 \\
\alpha^0 & \alpha^1 & \ldots & \alpha^{k-1} & 0 & 1 & \ldots & 0 \\
\alpha^0 & \alpha^2 & \ldots & \alpha^{2(k-1)} & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha^0 & \alpha^{t-1} & \ldots & \alpha^{(t-1)(k-1)} & 0 & 0 & \ldots & 1
\end{bmatrix}$$

$\alpha$: primitive element of $GF(2^r)$

Vandermonde matrix

• Using a Vandermonde matrix leads to simple encoder and decoder
• Not many constraints on code parameters in order to be MDS for small $t$
  – $t = 2$ or $3$ requires $k \leq 2^r - 1$
  – $t = 4$ requires $k \leq 27$ over $GF(2^8)$; $k \leq 67$ over $GF(2^{10})$; …
**Parity Splitting for Local Recovery**

MDS: every square submatrix of $\mathbf{A}$ is nonsingular

Any columns in $t'$ rows of $\mathbf{A}$ and $I_{t' \times t'}$ form the $H$ matrix of a $t'$-erasure-correcting MDS code

Divide the data into subsets and split the parities; $t' < t$ erasures are correctable by using local subsets

$$H = [A | I] = \begin{bmatrix} d_0 & d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & d_9 & d_{10} & d_{11} & p_{0} p_{1} p_{2} p_{3} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ \alpha^0 & \alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 & \alpha^7 & \alpha^8 & \alpha^9 & \alpha^{10} & \alpha^{11} & 0 & 1 & 0 & 0 \\ \alpha^0 & \alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 & \alpha^{10} & \alpha^{12} & \alpha^{14} & \alpha^{16} & \alpha^{18} & \alpha^{20} & \alpha^{22} & 0 & 0 & 1 & 0 \\ \alpha^0 & \alpha^3 & \alpha^6 & \alpha^9 & \alpha^{12} & \alpha^{15} & \alpha^{18} & \alpha^{21} & \alpha^{24} & \alpha^{27} & \alpha^{30} & \alpha^{33} & 0 & 0 & 0 & 1 \end{bmatrix}$$

- MDS: every square submatrix of $\mathbf{A}$ is nonsingular.
- Any columns in $t'$ rows of $\mathbf{A}$ and $I_{t' \times t'}$ form the $H$ matrix of a $t'$-erasure-correcting MDS code.
- Divide the data into subsets and split the parities; $t' < t$ erasures are correctable by using local subsets.

```
p_0
  p_{0,0} p_{0,1}
  p_{0,0,0} p_{0,0,1} p_{0,1,0} p_{0,1,1}
  p_{0,0,0,0} p_{0,0,0,1} p_{0,0,1,0} p_{0,0,1,1}
p_0,1
  p_{1,0} p_{1,1}
  p_{1,0,0} p_{1,0,1} p_{1,1,0} p_{1,1,1}
p_1
  p_{1,0,0} p_{1,0,1} p_{1,1,0} p_{1,1,1}
p_2
  p_{2,0} p_{2,1}
  p_{2,0,0} p_{2,0,1} p_{2,1,0} p_{2,1,1}

parent = child1 XOR child2
```
Examples

(i)  

(ii)  

(iii)  

(iv)  

(v)  

- Locality is defined as the number of symbols to access for recovery.
- Optimal subset division and parity splitting are dependent on erasure patterns & probabilities.

*(a,b,...) denote that the numbers of erasures in the last-level subsets are a,b,...

<table>
<thead>
<tr>
<th># of parities</th>
<th>1 erasure</th>
<th>2 erasures</th>
<th>3 erasures</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) 3</td>
<td>k</td>
<td>k</td>
<td>k</td>
</tr>
<tr>
<td>(ii) 5</td>
<td>k/3</td>
<td>2k/3 (1,1)</td>
<td>k</td>
</tr>
<tr>
<td>(iii) 5</td>
<td>k/2</td>
<td>k/2 (2,0)</td>
<td>k</td>
</tr>
<tr>
<td>(iv) 7</td>
<td>k/4</td>
<td>k/2</td>
<td>k/2 (1,1), 2k/3 (1,1,0)</td>
</tr>
<tr>
<td>(v) 6</td>
<td>k/3</td>
<td>k/3 (0,0,2)</td>
<td>2k/3 (2,1,0)</td>
</tr>
</tbody>
</table>

= data symbols
Encoder Hardware Architecture

With Vandermonde matrix

\[ p_i = \sum_{j=0}^{k-1} \alpha^i d_j = (\cdots ((d_{k-1} \alpha^i + d_{k-2}) \alpha^i + d_{k-3}) \cdots) \alpha^i + d_0 \]

Parity splitting

\[ p_{i0} = \sum_{j=0}^{k/2-1} \alpha^i d_j \]
\[ p_{i1} = \sum_{j=k/2}^{k-1} \alpha^i d_j \]

\[ p'_{i1} = \sum_{j=0}^{k/2-1} \alpha^{i+j+k/2} \]

\[ = p_{i1} / (\alpha^{i+k/2}) \]

- Split parities are derived by resetting the register when the data symbols of the next subset arrives
- The constant scalar is multiplied back before the split parities are added up to derive the parent parity if needed in the decoder
Decoding Formulas for $t=4$

- erasure indices: $w, x, y, z$

\[ d_w \alpha^w + d_x \alpha^x + d_y \alpha^y + d_z \alpha^z = p_i + \sum_{j \neq w, x, y, z} \alpha^j d_j = q_i \]

Solve linear equations:

\[ d_z = \frac{\alpha^{w+x+y} q_0 + (\alpha^w + \alpha^x + \alpha^y)q_1 + (\alpha^w + \alpha^x + \alpha^y)q_2 + q_3}{(\alpha^w + \alpha^z)(\alpha^x + \alpha^z)(\alpha^y + \alpha^z)} \]

\[ d_y = \frac{\alpha^{w+x} q_0 + (\alpha^w + \alpha^x)q_1 + q_2 + (\alpha^w + \alpha^x)(\alpha^x + \alpha^z)d_z}{(\alpha^w + \alpha^y)(\alpha^x + \alpha^y)} \]

\[ d_x = \frac{\alpha^w q_0 + q_1 + (\alpha^w + \alpha^y)d_y}{(\alpha^w + \alpha^x)} \]

\[ d_w = q_0 + d_x + d_y + d_z \]

- Formulas would be more complex if a non-Vandermonde matrix is used.
- Complexity is lower than that of Berlekamp-Massey RS decoder.
Decoder Hardware Architecture for $t=4$

- Single-erasure pre-correction is done first
- Overhead compared to non-local decoder is less than 15%
Comparisons with Existing Codes with Parity Manipulation

- Microsoft Azure locally recoverable code is a special case that splits only $p_0$
- Pyramid codes split parities and divide data sets in a similar way
  - Use existing MDS codes, whose systematic H generally has irregular entries
  - Has higher hardware complexity
- Codes with sparse parity columns in systematic generator matrix covering overlapping message symbols
  - Inferior locality for recovering data symbols